Track Fitting in the STAR Detector

using the Kalman Filter Method

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Abstract

This note is based on the work of D. Liko described in STAR Note 87. It is an overview of the *Kalman filter* code as currently implemented in STAR. The present code containins some modifications to the original code developed by D. Liko. These consist of a modified coordinate system, a more complete multiple scattering calculation, and corrections for energy loss in a medium.

1 Introduction

The *Kalman Filter* method was adapted for track fitting in three dimensions as a refit procedure after tracking was already done. The advantage of this type is an approach is that the contributions of multiple scattering and energy loss can easily be accommodated as part of the fitting procedure. With a reasonably good initial approximation of the track trajectories from the tracking methods already available the refit is quite fast and accurate. This note briefly describes the *Kalman filter* method and its application in STAR. For a more detailed discussion of the *Kalman filter* method see the references at the end of this note.

2 Kalman Filter

The *Kalman Filter*, a method derived from system theory, has been proposed as a novel approach to the reconstruction of charged tracks [1].

In the reconstruction of charged tracks it combines the advantages of former methods, especially the track fit with the breakpoint method [2], and adds a new concept for identification of outliers and further advantages [8]. Since it is not necessary to introduce additional parameters to describe multiple coulomb scattering, the concept of the reconstruction of charged tracks within a detector is simplified.

A vertex fit by the mean of the *Kalman filter* is equivalent to methods based on LSM estimators. Again an advantage of the *Kalman filter* method is an easy and fast method of outlier identification, that allows to reconstruct a primary vertex in the presence of a large number of tracks. At the present time the *Kalman filter* method for fitting the primary vertex has not been implemented in STAR.

Both algorithms have been already implemented and studied in both fixed target [3, 4] and other collider experiments [6, 7].

2.1 Track fitting

To reconstruct a charged track, usually a so-called *global* algorithm is used. A 5-dimensional parameter vector, at a certain surface within the detector is used to describe the track. Deviations of the track path due to multiple scattering can be described by the introduction of further parameters. These parameters add no additional degrees of freedom to the fit, as they are supplemented by measurements of the deviation with mean 0 and the usual error derived from the *Moliere* formula [2, 5]. The estimation of the parameters is usually derived by a Least Squares Method (LSM).

In *local* methods, as the *Kalman filter* and others describe the path of the particle by a set of 5-dimensional parameter vectors at all suitable surfaces within the detector. Generally, one can distinguish *active* surfaces, as a silicon detector board, and *passive* surfaces, as the beam pipe or a wall of a detector.

In the *Kalman filter* these parameter vectors $\mathbf{p}_i$ are linked by a set of so-called *system equations*:

$$\mathbf{p}_{i+1} = \mathbf{f}_i(\mathbf{p}_i) + \epsilon_i \tag{1}$$

with $i = 1 \ldots n$ denoting the various detector surfaces. The random quantity ϵ_i describes the *system noise*, given by multiple coulomb scattering processes and has again an expectation value of 0 and a variance given by the *Moliere* formula.

A further set of equations, the *measurement* equations, connect these parameter vectors with the individual detector measurements $\mathbf{m}_i$:

$$\mathbf{m}_i = \mathbf{h}_i(\mathbf{p}_i) + \delta_i \tag{2}$$

The random quantity δ_i describes the *measurement noise*, with an expectation value of 0 and a variance given by the detector resolution.

The fit itself is separated into two stages. During the first stage the parameter vector $\mathbf{p}_i$ from a surface $i-1$ is propagated to surface i including the associated covariance matrix. An additional term describing multiple coulomb scattering within the traversed material is added to the covariance matrix. The energy loss in the material is accounted by the change of the curvature term at each step. This gives $\mathbf{p}_i^{i-1}$, the *predicted* value at surface i with the knowledge of $i-1$ detectors. In the next step, the so-called *filter* step, the measurement at surface i is added with a usual weighted mean. When the last detector, n, is reached, the full information of the setup is used and the calculated parameter vector is an optimal estimate of the track.

In the second stage, the *smoother* this information is propagated by mean of the *Kalman smoother* to the $n-1$ other surfaces, giving $\mathbf{p}_i^n$, the parameter vector at all surfaces i, with the knowledge of all detectors.

It is important to mention that all these calculations are done in the context of a LSM. In the ideal case LSM is known to have very appealing properties, but its performance may be strongly degraded by the presence of wrong measurements, so-called outliers (e.g. wrongly associated hits, etc.).

During the smoother a χ^2-quantity for each measurement is calculated, that can be used to identify outliers. Note that this quantity has a significant advantage to a similar quantity, calculated during the *filter* steps, as now the full knowledge of all detectors is used.

Passive surfaces, as detector walls, can be trivially added as surfaces without measurements, where only the increase of the covariance matrix due to multiple scattering is important.

For a more detailed description of the algorithm see for example [9, 10].

3 Track fitting for STAR

The current implementation of the *Kalman filter* code is for the so called year 1 geometry (TPC tracking detector only). The material for multiple scattering is specified for a continuous medium (P10 gas) and the barrel of the inner field cage as additons of radiation length specified in the code. The energy loss is obtained from GEANT tables which are calculated from the STAR geometry.

The TPC hit positions and errors are used as input the the *Kalman filter* fit on a track by track basis. The variance of the hit errors (or the correlation) is assumed to be zero. This does not preclude the use of the finite correlations if they are available for each hit.

3.1 Coordinate System

The coordinate system for the fit is based on the STAR coordinate sytem with slight modification for convenience of the fit.

As the track fitting within magnetic fields is a nonlinear problem, the choice of a suited coordinate system, that is well described by local expansion around some reference parameters, is important. This question is discussed in detail in reference [2]. Solving the equation of motion within a magnetic fields leads to a set of 5 parameters at a given reference surface, in this case the radius R of a detector surface. Following parameterization is used:

$(R\phi)$ ...Radial distance from the beam line times the azimuthal angle of the position of track at the reference surface (or arclength);

z ... The position of the track at the reference surface in z;

$\tan(\lambda)$ ... The tangent of the dip angle of the track at the reference surface;

Ψ ... The azimuthal angle of the track at the reference surface;

(q/p_t) ... Charge (+1 or -1) divided by the transverse momentum.

The choice of these parameters is driven by the track model. As the magnetic field is nearly homogenous, it is in this case a helix. The tracking routines, that are used to propagate the parameters from one detector surface to next are described in appendix A.1. These routines calculate also the corresponding derivative matrices used to propagate the covariance matrix.

3.2 Track fitting in the STAR TPC

Input for the track fit is the set of associated hits from the pattern recognition. Together with these hits, the pattern recognition provides also a first estimate of the track parameters. Such information is not really necessary, but simplifies the fitting procedure, as the track model can be evaluated along this track.

It is necessary to define a direction for the various steps. As the track density at the outer wall of the TPC is assumed to be lower, the fit is initialized at the position of the hit with the largest distance from the vertex. During the first filter steps, enough information is accumulated to provide a stable extrapolation.

This property of the *Kalman filter* has been found to be a disadvantage, especially in application where the algorithm is also used for pattern recognition. It may then be necessary to try various alternatives, until a real track can be reconstructed. In an application, where the *Kalman filter* is only used to fit the track, it is usually no problem.

Track fitting is done in two passes: filtering followed by smoothing. The filtering pass is done starting in steps from the hit at the largest radius. The smoothing pass is done in steps from the hit at the smallest radius. The third pass is for evaluation of the total chisquare and the tracks parameters.

Outliers are identified and removed during the first pass. A very crude cut removes obvious outliers during the filter phase. Only in the smoother the full information of all hits is available and outliers are removed after a more severe cut.

If outliers are identified and removed in the smoother, the optimal estimate of the track parameters is only available at the position of the first hit. The easiest way to provide this information also at the position of the hit nearest to the inner wall of the TPC, is to reevaluate the fit without further cuts.

The final result is given by the estimate of the track parameters at the position of the innermost hit. This allows optimal extrapolation of the track into the vertex region. If it is necessary to extrapolate tracks also to the outer region of the detector (calorimeters) it is an advantage to use the estimation of the track at the position of the last hit. As the *Kalman filter* can follow the real path of a track, this would provide more precise information, than an extrapolation from the innermost hit.

3.3 Track fitting by adding the SVT hits

The current implementation of the *Kalman filter* code for STAR deals only with year 1 geometry, i.e. it does not yet have the code to add the SVT and SSD hits properly. In principle, this should be straightforward when code to deal with the SVT geometry is implemented.

Briefly, the procedure would be to add vertex detector hits when the track is extrapolated to the SVT layers. Multiple scattering can be evaluated along the track and the covariance matrix accordingly modified (passive surfaces). Hits would then be added by subsequent filter steps.

A Short description of the routines

All programs are in Fortran 77. This is legacy code!

One part of the files are of general use, e.g. the track model and *Kalman filter* matrix routines:

- `ktrack.F` ... Track fit;
- `helix.F` ... Tracking routines;

- `kalex.F` ... Kalman extrapolator routine;

The second set consists of the routines used to interface to the STAR code:

- `tpcfit.F` ... TPC track fit;

Some of the routines are briefly described below.

The track information is stored in an vector of length 23 by the main procesing routine FITTER. This routine is called for each track. The track is described by 6 entries for the track parameters - R, $R\phi$, z, $\tan(\lambda)$, Ψ, q/p_t - followed by the corresponding covariance matrix (15 entries), the χ^2 and the number degrees of freedom of the fit.

A.1 HELIX

This is the library to perform the calculations for the track model. Since there is nearly a homogeneous magnetic field, these routines solve the intersection between the extrapolation of a helix and a cylinder with given radius.

```
SUBROUTINE VXBTRK(RIN,PIN,ROU,POU,IERR)
```

Input:

`RIN` ... First reference surface

`PIN(5)` ... Parameter vector at first reference surface

`ROU` ... Second reference surface

Output:

`POU(5)` ... Momentum vector at second reference surface

`IERR` ... Error Code

Purpose:
A track described by a 5-dimensional parameter vector `PIN` at reference `RIN` is extrapolated to the reference surface `ROU` giving `POU` (see also section 3.1).

```
SUBROUTINE VXBTRD(RIN,PIN,ROU,POU,DER,IERR)
```

Input:

`RIN` ... First reference surface

`PIN(5)` ... Parameter vector at first reference surface

`ROU` ... Second reference surface

Output:

`POU(5)` ... Momentum vector at second reference surface

`DER(5,5)` ... Derivative matrix

`IERR` ... Error Code

Purpose:
A track described by a 5-dimensional parameter vector `PIN` at reference `RIN` is extrapolated to the reference surface `ROU` giving `POU` (see also section 3.1). In addition, the corresponding derivative matrix (e.g. ∂`POU`/∂`PIN`) is also calculated.

A.2 TPCFIT

These are the routines, used to reconstruct the track within the TPC. It uses `ktrack.F` routines to fit the tracks and `helix.F` routines to calculate the track model. Outlier rejection is part of the fitting procedure (see section 3.2.

```
SUBROUTINE TPCINI(HYPOID,OUTFIL,OUTSMO)
```

Input:

`HYPOID` ... GEANT particle ID used in energy loss and multiple scattering calculations

`OUTFIL` ... Outlier Confidence Level Cut during Filter

`OUTSMO` ... Outlier Confidence Level Cut during Smoother

Purpose:
Initialization of the TPC track fit; once at the start of the program or once per track, depending on conditions

```
SUBROUTINE TPCTRK(NEVT,NTRK,TRKGEN,TRKREF,NTPC,TPC,TPCCH2,TRKTPC,IRET)
```

Input:

`NEVT` ... Event Number

`NTRK` ... Track Number

`TRKGEN(6)` ... Track generation parameter at position of first hit (R, $R\phi$, z, $\tan(\lambda)$, Ψ, q/p_t);

`TRKREF(6)` ... Reference track at position of last hit;

`NTPC` ... Number of TPC hits

`TPC(6,NTPC` ... TPC hits (R, $R\phi$, z, $\mathrm{var}(R\phi)$, $\mathrm{cov}(R\phi, z)$, $\mathrm{var}(z)$);

Output:

`TPCCH2(NTPC)` ... χ^2-contribution of corresponding hit; for Outliers -1;

`TRKTPC(23)` ... Reconstructed Track;

`IRET` ... Return Code;

Purpose:
The given hits within the TPC are fitted.

A.3 KALEX

This routine is to be called for extrapolating tracks to an inactive detector surface from a reference point on a fitted track to a cylindrical surface R. The track vector at the new surface and the covariance vector at that surface is returned.

```
SUBROUTINE KALEX(STEP,RL,PL,CL,R,P,C,IDGEANT,IRET)
```

Input:

`STEP` ... extrapolation step size in cm.

`RL` ... current radius

`PL(5)` ... current reference track parameter vector

`CL(15)` ... current cov. matrix

`R` ... new radius (¡=RL)

`IDGEANT` ... GEANT particle ID hypothesis

Output:

`P(5)` ... new reference track parameter vector

`C(15)` ... new cov. matrix

`IRET` ... return code (=0 for success)

Purpose:
A track extrapolation, including covariance matrix, using the Kalman filter method. A call to TPCINI (see above) is required before KALEX.

References

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